26.

$f(t) = \frac{2}{1-t^2}$ $\quad g(x) = \int_0^x f(t)\, dt$

a) $f(t) =$

Feb 5-9:22 AM

43 $\quad f(x) = \frac{\sin x}{x}$

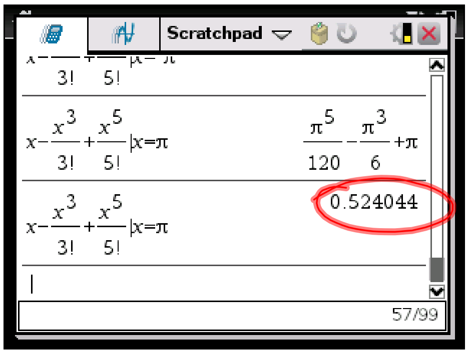
Feb 5-9:35 AM

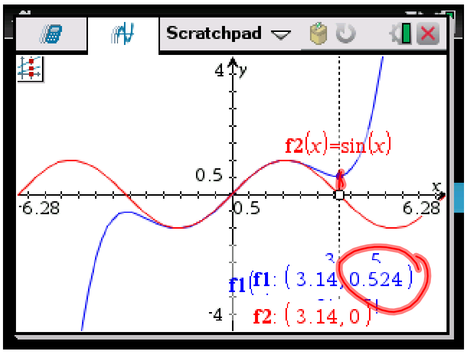
9.3 Taylor series with remainder

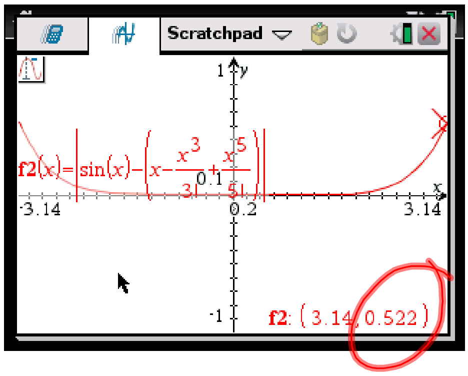
What is the fifth order Maclaurin series for f(x)=sin(x)? What is the maximum error when approximating sin(x) on $[-\pi,\pi]$

Solve graphically and numerically

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

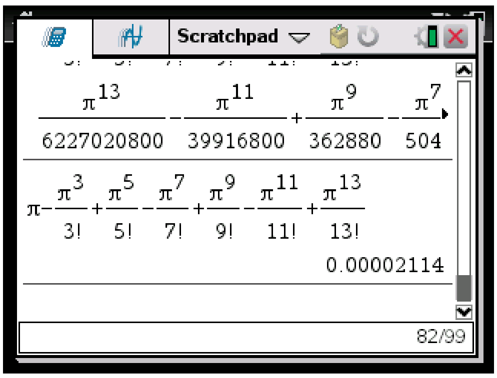




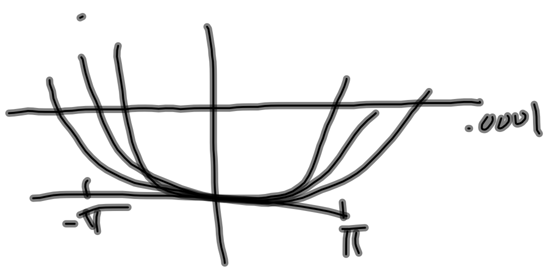


Jan 31-5:30 PM

How many terms are needed in the Maclaurin series for sin(x) in order to approximate sin(x) within .0001 on the interval $[-\pi,\pi]$

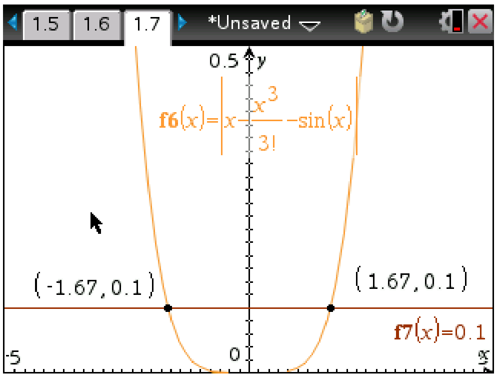


trial & error



Jan 31-6:00 PM

On what interval does the third order Maclaurin series approximate sin(x) within .01?



Jan 31-6:02 PM

Taylor's Remainder Estimation Theorem

$$f(x) \approx f(a) + f'(a)(x-a) \ldots \frac{f^{n}(a)}{n!}(x-a)^{n} + \epsilon$$

$$\epsilon \leq \left| \frac{M (x-a)^{n+1}}{(n+1)!} \right|$$

M is max of $f^{n+1}(x)$

Jan 31-6:03 PM

The approximation $\ln(1+x) \approx x - \frac{x^2}{2}$ is used when x is small.

Use the Remainder Estimation Theorem to get a bound for the maximum error when $|x| \le .01$ Support the answer graphically.

$-.01 \le x \le .01$

$\epsilon \le \left| \frac{M \cdot x^3}{3!} \right|$ M is max $f^3(x)$

$f(x) = \ln(1+x)$

$f'(x) = \frac{1}{1+x}$

$f'' = \frac{-1}{(1+x)^2}$

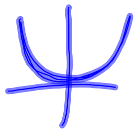
$f^3 = \frac{2}{(1+x)^3}$

$M = \frac{2}{(1+-.01)^3} = \frac{2}{.99^3}$

$= 2.061$

$\epsilon \le \left| \frac{2.061\, x^3}{3!} \right|$

$\epsilon \le \left| \frac{2.061 \cdot .01^3}{3!} \right|$



$\epsilon \le 3.435 \times 10^{-7}$

Jan 31-6:07 PM